Stability of gravity-scalar systems for domain-wall models with a soft wall

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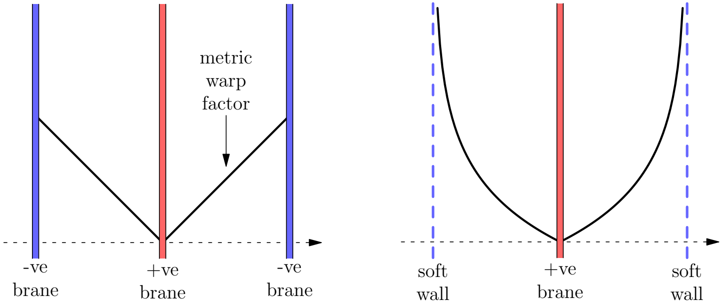
Compact extra dimensions

Physics beyond the standard model: *compact* extra dimensions.



Randall-Sundrum hard wall → RS soft-wall models.

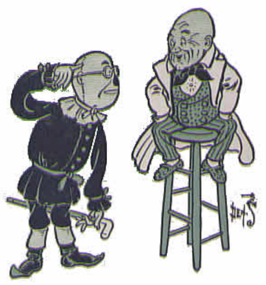
Original soft-wall motivation: AdS/QCD and linear Regge trajectories.

Karch, Katz, Son & Stephanov, PRD74, 015005 (2006)

Now exist early BSM models.

Batell & Gherghetta, PRD78, 026002 (2008), Falkowski & Perez-Victoria, JHEP 12, 107 (2008),
Batell, Gherghetta & Sword, PRD78, 116011 (2008), Cabrer, von Gersdorff & Quiros, arXiv:0907.5361,
Aybat & Santiago, PRD80, 035005 (2009).

Branes: you don't need them!

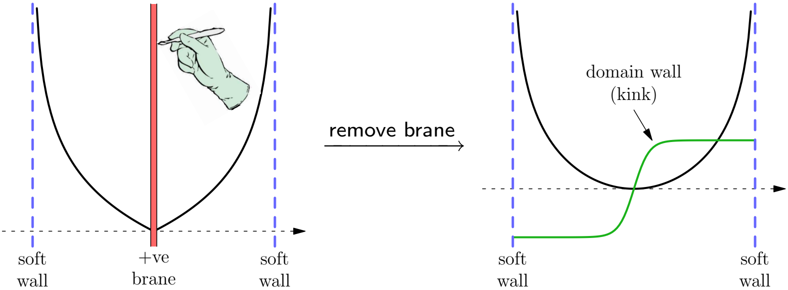


"Can't you give me brains?" asked the Scarecrow.

"You don't need them!" replied the Wizard.

Our aim: replace the brane with a domain wall.
→ *Must ensure stability*.

Work based on arXiv:1006.2827, with Mert Aybat.



General set-up; background configuration

General framework: GR with N scalar fields:

$$\mathcal{S} = \int d^4x\, dy \left[\sqrt{-g}\left(M^3 R + \mathcal{L}_{\text{matter}}\right) - \sqrt{-g_4}\lambda\right] ,$$

with

$$\mathcal{L}_{\text{matter}} = -\frac{1}{2}\sum_i g^{MN}\partial_M\Phi_i\partial_N\Phi_i - V(\{\Phi_i\}) ,$$

$$\lambda = \lambda(\{\Phi_i\}) = \sum_\alpha \lambda_\alpha(\{\Phi_i\})\delta(y - y_\alpha) .$$

Background ansatz: $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$, $\Phi_i(x^\mu, y) = \phi_i(y)$.

Einstein's and Euler-Lagrange equations:

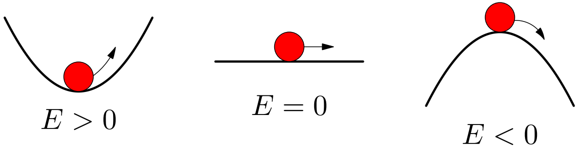
$$6M^3\sigma'' = \sum_i \phi_i'^2 + \lambda(\{\phi_j\}) , \quad 6M^3(\sigma'' - 4\sigma'^2) = 2V\left(\{\phi_j\}\right) + \lambda\left(\{\phi_j\}\right) ,$$

$$\phi_i'' - 4\sigma'\phi_i' - V_i(\{\phi_j\}) - \lambda_i(\{\phi_j\}) = 0 . \qquad (\sigma' = d\sigma/dy, V_i = \partial V/\partial\Phi_i \text{ etc.})$$

$\{V$, λ, integration constants$\}$ define a configuration.
Is it stable in the space of configurations?

Perturbative stability

Perturb the background configuration → eigenvalue problem.



Spin-0 and spin-2 perturbations described by:

$$ds^2 = e^{-2\sigma}\left[(1 - 2F(x^\mu, y)\eta_{\mu\nu} + h_{\mu\nu}(x^\mu, y)\right] dx^\mu dx^\nu + \left[1 + 4F(x^\mu, y)\right] dy^2,$$
$$\Phi_i(x^\mu, y) = \phi_i(y) + \varphi_i(x^\mu, y)\ .$$

(Axial gauge $h_{\mu 5} = 0$, transverse traceless $\partial^\mu h_{\mu\nu} = \eta^{\mu\nu} h_{\mu\nu} = 0$.)

Degrees of freedom:

- Spin-2: $h_{\mu\nu}$ decouples from F and φ_i. Is non-tachyonic. Has a zero mode (4D graviton). Known RS2 result.
- Spin-0: Non-trivial. Physical modes are mixtures of F and φ_i.

Spin-0 perturbations

For spin-0, the equations to solve are ($\Box = \partial^\mu \partial_\mu$):

$$6M^3(F' - 2\sigma' F) = \phi_i' \varphi_i \ ,$$
$$6M^3(-e^{2\sigma}\Box F - 2\sigma' F' + F'') = 2\phi_i' {\varphi_i}' + 2\ \lambda|_{\mathsf{bg}}\, F + \lambda_i|_{\mathsf{bg}}\, \varphi_i \ ,$$
$$e^{2\sigma}\Box\varphi_i + {\varphi_i}'' - 4\sigma' {\varphi_i}' - 6\phi_i' F' - (4V_i + 2\lambda_i)|_{\mathsf{bg}}\, F - (V_{ij} + \lambda_{ij})|_{\mathsf{bg}}\, \varphi_j = 0.$$

We can do it! Go to conformal coordinates ($dy = e^{-\sigma} dz$), rescale fields:

$$F(y) = \frac{1}{\sqrt{12}} e^{3\sigma/2} \chi(z(y)) \ , \qquad \varphi_i(y) = M^{3/2} e^{3\sigma/2} \psi_i(z(y)) \ .$$

Massage into a familiar form:

$$-\chi'' + (\mathcal{V}_{00} + \mathcal{B}_{00})\chi + (\mathcal{V}_{0i} + \mathcal{B}_{0i})\psi_i = \Box\chi \ ,$$
$$-{\psi_i}'' + (\mathcal{V}_{0i} + \mathcal{B}_{0i})\chi + (\mathcal{V}_{ij} + \mathcal{B}_{ij})\psi_j = \Box\psi_i \ .$$

where $\mathcal{B}_{mn}$ contain brane-only terms, and

$$\mathcal{V}_{00} = \frac{9}{4}\sigma'^2 + \frac{5}{2}\sigma'' \ , \qquad \mathcal{V}_{0i} = \frac{2}{\sqrt{3M^3}} \phi_i'' \ ,$$
$$\mathcal{V}_{ij} = \left(\frac{9}{4}\sigma'^2 - \frac{3}{2}\sigma''\right)\delta_{ij} + \frac{1}{M^3}\phi_i'\phi_j' + e^{-2\sigma}\ V_{ij}|_{\mathsf{bg}} \ .$$

Fake supergravity

We now specialise to potentials $V(\{\Phi_i\})$ generated using the fake supergravity approach (no branes):

$$V(\{\Phi_i\}) = \sum_i \frac{1}{2} \left[W_i(\{\Phi_i\})\right]^2 - \frac{1}{3M^3} \left[W(\{\Phi_i\})\right]^2 \ .$$

DeWolfe, Freedman, Gubser & Karch, PRD62, 046008 (2000)
Freedman, Nunez, Schnabl & Skenderis, PRD69, 104027 (2004)

For any W, solutions to Einstein's and Euler-Lagrange are:

$$\sigma' = \tfrac{1}{6M^3} W(\{\phi_i\}) \,,$$
$$\phi_i' = W_i(\{\phi_i\}) \,.$$

This is a set of *first order* equations:

- W encodes for V and half of the integration constants.
- Can take $\sigma(y_0) = 0$ without loss of generality.
- Have N integration constants left: set of values $\{\phi_i(y_0)\}$.

$\{W, \phi_i(y_0)\}$ uniquely define a configuration. Is it stable?

Perturbative stability in fake SUGRA approach

Recall, for general V perturbations obey

$$-\begin{pmatrix}\chi\\ \psi_i\end{pmatrix}'' + \begin{pmatrix}\mathcal{V}_{00} & \mathcal{V}_{0j}\\ \mathcal{V}_{0i} & \mathcal{V}_{ij}\end{pmatrix}\begin{pmatrix}\chi\\ \psi_j\end{pmatrix} = \Box\begin{pmatrix}\chi\\ \psi_i\end{pmatrix}\,.$$

Using fake SUGRA, we find that $\mathcal{V} = \mathcal{S}^2 + \mathcal{S}'$, where

$$\mathcal{S} = e^{-\sigma}\begin{pmatrix}\frac{1}{12M^3}W & \frac{1}{\sqrt{3M^3}}W_j\\ \frac{1}{\sqrt{3M^3}}W_i & -\frac{1}{4M^3}\delta_{ij}W + W_{ij}\end{pmatrix}\Bigg|_{\text{bg}}\,.$$

Write $\Psi = (\chi, \psi_i)^T$. Perturbations obey $(\partial_z + \mathcal{S})(-\partial_z + \mathcal{S})\Psi = \Box\Psi$.

Fourier transform on x^μ, multiply by $\Psi^\dagger$ on left and integrate:

$$\int |(-\partial_z + \mathcal{S})\Psi|^2\, dz + \text{(boundary terms)} = E\int |\Psi|^2 dz\,.$$

Boundary terms vanish for warped metric. **We find $E \geq 0$.**

What about $E = 0$? Such modes can correspond to changes in the size of the compact extra dimension.

$N = 1$ systems

With $N = 1$ scalar, can use constraint Einstein equation to eliminate ψ_1 in terms of χ. Then redefine $\chi = \mathcal{S}_{01} g$.

Schrödinger-like equation for g is factorisable:

$$(\partial_z - \mathcal{S}_{11})(-\partial_z - \mathcal{S}_{11})g + \mathcal{S}_{01}^2 g = Eg \, .$$

(Recall: $\mathcal{S}_{01} = \frac{1}{\sqrt{3M^3}} W_1$, $\mathcal{S}_{11} = -\frac{1}{4M^3} W + W_{11}$.)

Multiply by g and integrate:

$$\int |(-\partial_z - \mathcal{S}_{11})g|^2 \, dz + \int |\mathcal{S}_{01} g|^2 + \text{(boundary terms)} = E \int |g|^2 dz \, .$$

For warped metric, boundary terms vanish. Then for $E = 0$ require:

- $(-\partial_z - \mathcal{S}_{11})g = 0$.
- $\mathcal{S}_{01} g = 0$.

Systems with $N = 1$ do not have a zero mode.

$N = 2$ and the zero-mode theorem

For $N = 2$ there may or may not be a zero mode.

Theorem: *For a system of definite parity with N scalar fields that couple to gravity, the number of independent normalisable zero modes with $E = 0$ is at most equal to the number of fields whose background solutions are even.*

Proof: If a zero mode exists, adding it to the background takes you to another background, generated using the same superpotential but with different integration constants. (Recall: $\sigma' = \frac{1}{6M^3}W$, $\phi_i' = W_i$.)

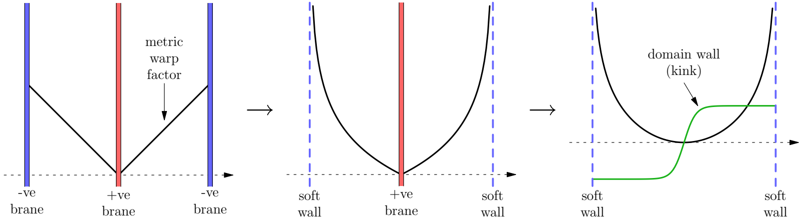
Zero modes $\leftrightarrow$ available integration constants.

No integration constants $\implies$ no zero modes.

Look at some examples with $N = 2$ scalars.

Domain-wall models with a soft wall

Branes, soft walls and domain walls.



Our domain-wall model is specified by

$$\mathcal{S} = \int d^4x\, dy \sqrt{-g} \left[M^3 R - \sum_{i=1,2} \tfrac{1}{2} g^{MN} \partial_M \Phi_i \partial_N \Phi_i - \sum_{i=1,2} \tfrac{1}{2} W_i^2 + \tfrac{1}{3M^3} W^2 \right].$$

- Φ_1, the dilaton: diverges at finite y to create a soft wall.
- Φ_2, the domain wall: has a kink profile to provide energy density at the origin.
- σ, the warp factor: diverges at finite y.

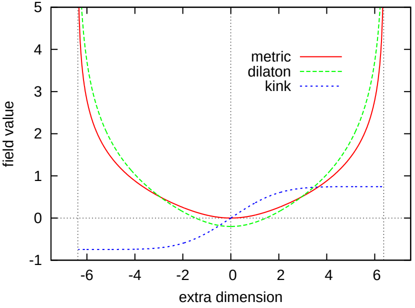
Example $N = 2$ even system — unstable

Φ_1 is even, Φ_2 is odd, σ is even.

Superpotential: $W(\Phi_1, \Phi_2) = e^{\nu\Phi_1}\left(a\Phi_2 - b\Phi_2^3\right)$

Φ_1 is even:

- choice of integration constant
- corresponding zero mode
- size of extra dimension is *not* fixed



$(\nu = 1.4,\ a = 0.5,\ b = 0.3)$

For this case, we have found the explicit solution for the zero mode.

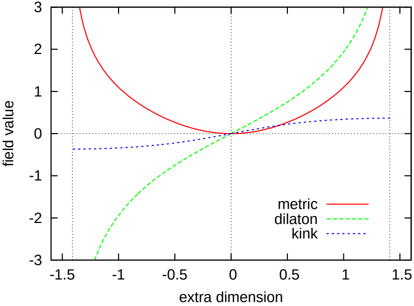
Example $N = 2$ odd system — stable

Φ_1 is odd, Φ_2 is odd, σ is even.

Superpotential: $W(\Phi_1, \Phi_2) = \alpha \sinh(\nu \Phi_1) + \left(a\Phi_2 - b\Phi_2^3\right)$

Φ_1 and Φ_2 are odd:

- no integration constants to choose
- background solution is unique
- no zero modes
- size of extra dimension fixed by parameters in W



$(\alpha = 1,\ \nu = 1.4,\ a = 0.5,\ b = 0.3)$

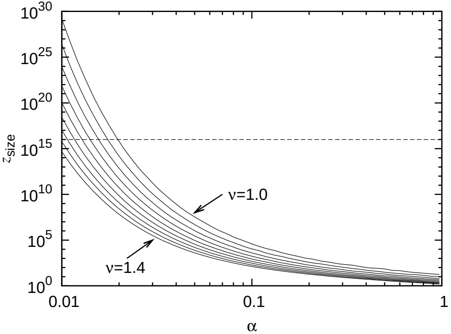
An example of a domain-wall model with a soft wall, that stabilises the size of a compact extra dimension.

The hierarchy problem in domain-wall soft-wall models

Superpotential: $W(\Phi_1, \Phi_2) = \alpha \sinh(\nu \Phi_1) + \left(a\Phi_2 - b\Phi_2^3\right)$

Characteristic scale: z_{size} = size of extra dimension in conformal coordinates.

- Bulk fields KK mass scale: $m_{\text{KK}} \sim z_{\text{size}}^{-1}$.
- Hierarchy problem solved if $z_{\text{size}} \sim 10^{16}$ for $\mathcal{O}(1)$ model parameters.
- For $a = b = 1$, need $\alpha \simeq 0.02$ $\nu \simeq 1.0$.



$(\alpha = 0.01 \ldots 1.0,\ \nu = 1.0 \ldots 1.4,\ a = 1,\ b = 1)$

Application to AdS/QCD

Model QCD by a 5d theory.

Having a soft wall in the IR yields linear Regge trajectories: meson excitations $m_n^2 \sim n$.

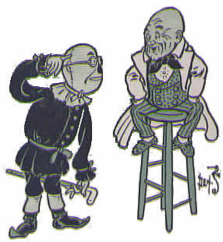
Dynamically generate the 5d background: use dilaton and tachyon.
Batell & Gherghetta, PRD78, 026002 (2008)

Scalar fluctuations correspond to glueball and scalar meson excitations.

Superpotential approach is common in literature. Using our results, can compute the scalar spectrum with multiple background fields.

Summary and future work

Branes: not necessary!



We can have a stable, compact extra dimension:

- Soft-wall at edge of space; replaces negative brane.
- Domain-wall at origin; replaces positive brane.
- Additional scalar (dilaton) cuts off space.

Lesson: using fake SUGRA, need definite parity and all scalars must be odd to eliminate zero modes.

Technical questions:

- No fake SUGRA: can we have even fields?
- Odd fields, but not definite parity: are there zero modes?

Most interesting questions:

- Can we solve the hierarchy problem? Improve upon $\alpha \sim 0.02$.
- Can we build a realistic standard model?